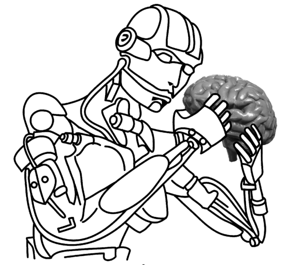
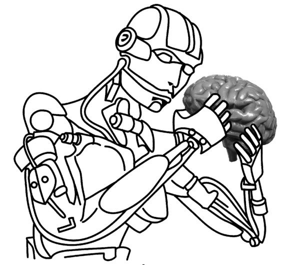
CS545—Contents XI



- Newton-Euler Method of Deriving Equations of Motion
  - Newton's Equation
  - Euler's Equation
  - The Newton-Euler Recursion
  - Automatic Generation of Equations of Motion
- Reading Assignment for Next Class
  - See http://www-clmc.usc.edu/~cs545

Newton’s & Euler’s Equations



- Newton’s Equation:
  - Expresses the force acting at the CM for an accelerated body

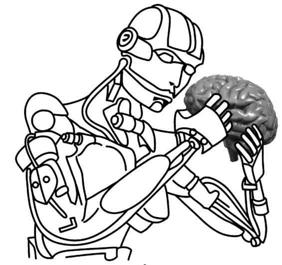
$$\mathbf{F} = m\ddot{\mathbf{p}}_{cm}$$

- Euler’s Equations
  - Expresses the torque acting on a rigid body given an angular velocity and angular acceleration

$$\tau = I^{cm}\dot{\omega} + \omega \times I^{cm}\omega$$

- Idea of Newton-Euler
  - Force and torque balance at every link
  - Recursion through all links
- What do we need?
  - Angular velocities and accelerations of each link
  - Linear velocities at each link

Computing Angular Velocities



- Start at joint 1, recursion to higher joints
- Angular Velocity of link i+1, expressed in base coordinates

$$\omega_i = \omega_{i-1} + \dot{\theta}_i z_{i-1}$$

- What happens to prismatic joints in this equation?
  - Nothing :-)

$$\omega_i = \omega_{i-1}$$

Computing Angular Accelerations

- Start at joint 1, recursion to higher joints
- Angular Acceleration of link i+1, expressed in coordinate system i+1 is obtained from differentiating the angular velocities

$$\omega_{i+1}^{i+1} = \mathbf{R}_i^{i+1}\omega_i^i + \dot{\theta}_{i+1} z_{i+1}^{i+1}$$

$$\dot{\omega}_{i+1}^{i+1} = \mathbf{R}_i^{i+1}\dot{\omega}_i^i + \mathbf{R}_i^{i+1}\omega_i^i \times \dot{\theta}_{i+1} z_{i+1}^{i+1} + \ddot{\theta}_{i+1} z_{i+1}^{i+1}$$

- For prismatic joints

$$\dot{\omega}_{i+1}^{i+1} = \mathbf{R}_i^{i+1}\dot{\omega}_i^i$$