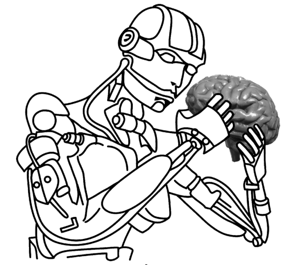
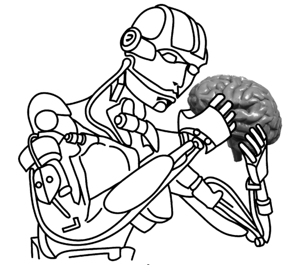
CS545—Contents XII



- Nonlinear Control
  - Joint space control
    - Decoupled control
      - PID control in joint space
    - Centralized control
      - Compute torque control
      - Inverse dynamics control
  - Operational space control
- Reading Assignment for Next Class
      - See http://www-clmc.usc.edu/~cs545

Two Control Spaces



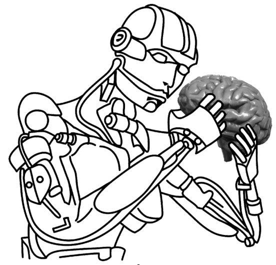
- Joint Space:
  - Solve two separate subproblems
    - Inverse kinematics to transform desired trajectories in operational space into joint space (including higher derivatives!)
    - Joint space controller tracks desired trajectories in joint space
    - Quality of control depends on quality of kinematics model (indirect control method)
- Operational Space:
    - Formulate the controller directly in operational space, e.g., a PD controller in Cartesian space.
    - Inverse kinematics is somehow included in the operational space controller
    - Quality of control is independent of kinematics if sensor measurements are taken directly in operational space (but this is rarely the case)
    - Controller can break if inverse kinematics is ill-defined.

Joint Space Control

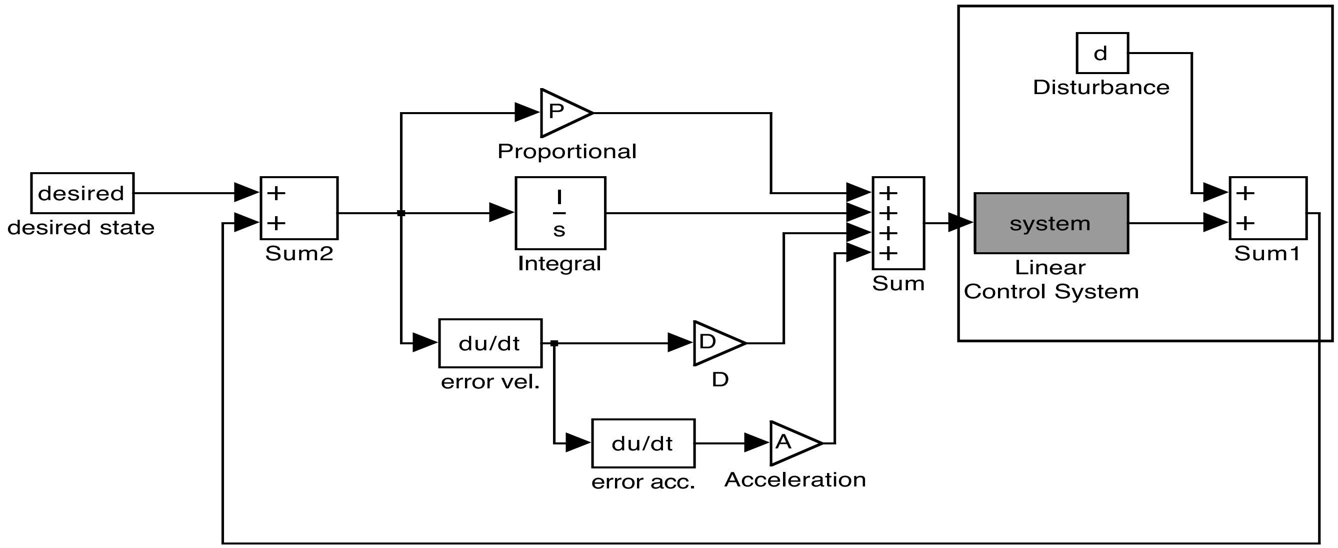
$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau$$

- Independent (de-centralized) Joint Space Control
  - Is appropriate when:
    - Coupling terms are negligible and can be treated as disturbances
    - Robot is really decoupled (i.e., like a set of 1DOF robots)
    - Gains can be chosen really high
    - No computational power exists
- Dependent (centralized) Joint Space Control
  - Is appropriate when:
    - Coupling terms cannot be neglected anymore

Independent Joint Space Control

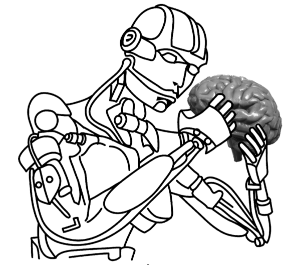


- Negative Feedback Control in Joint Space
  - Coupling terms are treated as disturbances

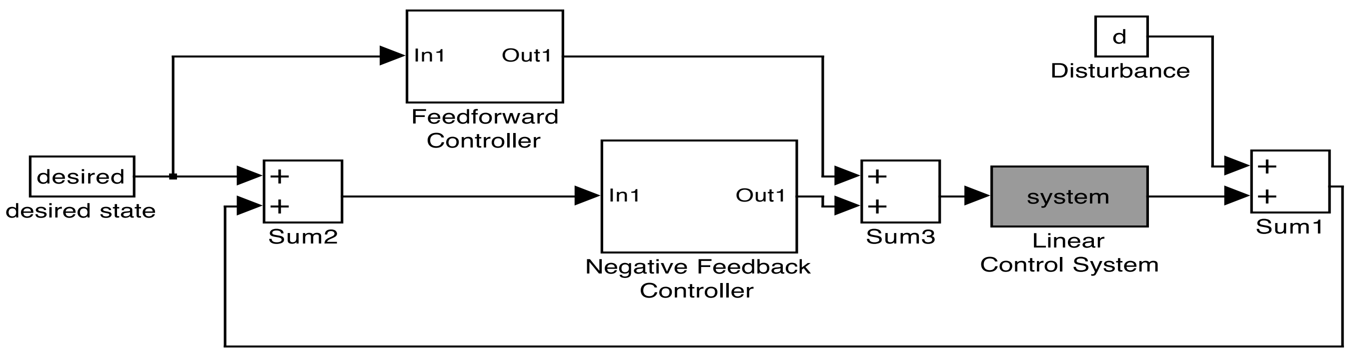


  - In order to deal with disturbances from coupling terms more effectively, an acceleration-based feedback term is often added (note that good acceleration signals are not easy to obtain from real sensors)

Independent Joint Space Control (cont'd)

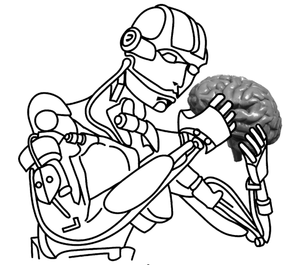


- Feedforward Compensation in Joint Space



- What is the right choice of a de-centralized feedforward controller?
  - Need de-centralized inertial, damping, and spring term

Computed Torque Feedforward Control

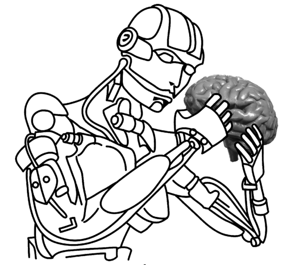


- A hybrid centralized–decentralized control strategy
  - Use de-centralized PID controller for stabilization
  - Use (centralized) inverse dynamics model to add feedforward command

$$
\begin{aligned}
&\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{G}(\mathbf{q})=\tau\\
&\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{G}(\mathbf{q})=\\
&\mathbf{B}(\mathbf{q}_d)\ddot{\mathbf{q}}_d+\mathbf{C}(\mathbf{q}_d,\dot{\mathbf{q}}_d)\dot{\mathbf{q}}_d+\mathbf{G}(\mathbf{q}_d)+\mathbf{K}_P(\mathbf{q}-\mathbf{q}_d)+\mathbf{K}_D(\dot{\mathbf{q}}-\dot{\mathbf{q}}_d)\\
&\text{In approximation:}\\
&\mathbf{B}(\mathbf{q})\ddot{\mathbf{e}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{e}}+\mathbf{G}(\mathbf{q})\mathbf{e}=\mathbf{K}_P\mathbf{e}+\mathbf{K}_D\dot{\mathbf{e}}\\
&\mathbf{B}(\mathbf{q})\ddot{\mathbf{e}}+\left(\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})-\mathbf{K}_D\right)\mathbf{e}+\left(\mathbf{G}(\mathbf{q})-\mathbf{K}_P\right)\mathbf{e}=0
\end{aligned}
$$

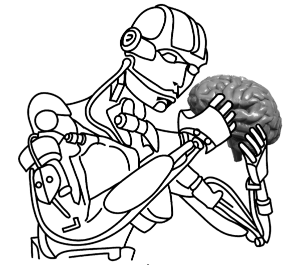
- The error dynamics forms a second order linear time variant dynamical system that can be stabilized with a suitable choice of the feedback gains (assuming the model is accurate)

Computed Torque Feedforward Control (cont'd)



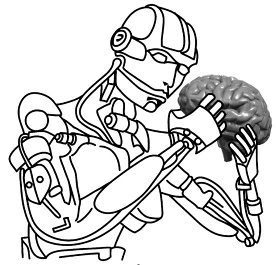
- Remarks:
  - Feedforward command is only based on DESIRED states
    - Higher derivatives of desired states are usually clean
  - Feedforward commands are not very accurate if the system deviates too much form the desired trajectory
  - Feedforward commands can be computed off-line if necessary (computational burden)

Nonlinear Centralized Control

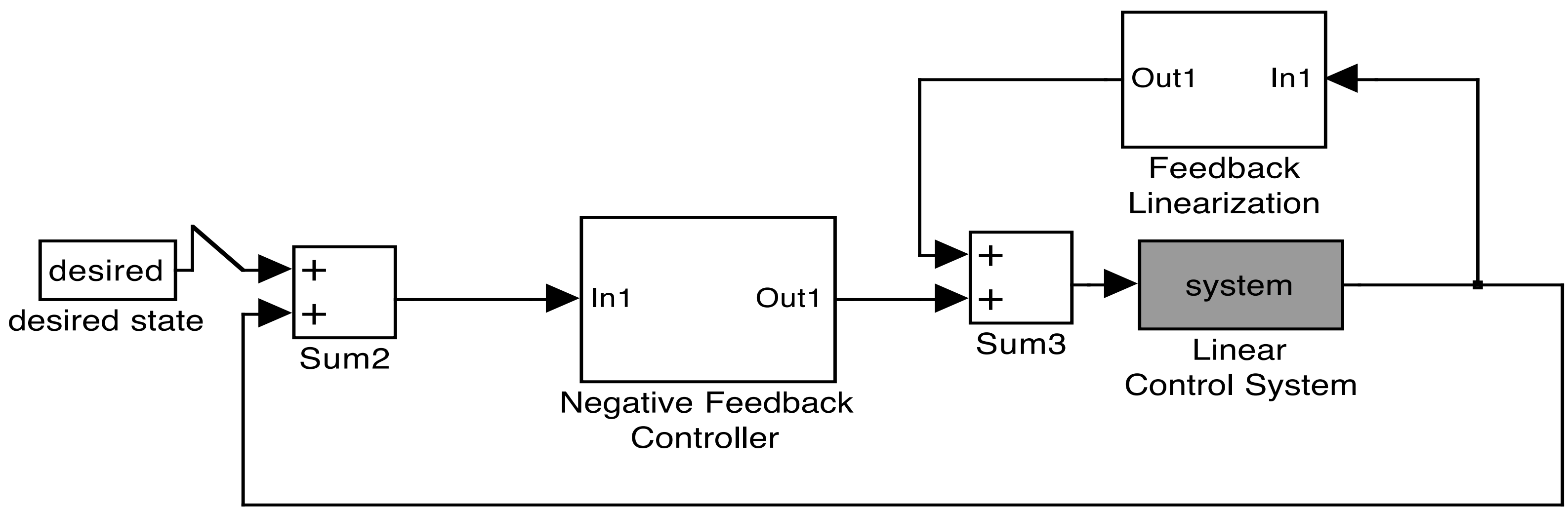


- Decentralized and hybrid approaches are not really proper centralized nonlinear controllers since the negative feedback control law assumes decoupling
- Goal: Develop a nonlinear negative feedback controller
- Most common approach: Feedback linearization

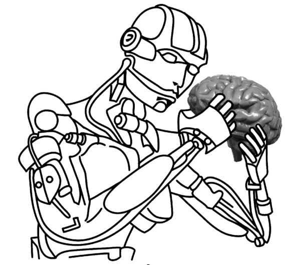
A Simple Example of a Nonlinear Control Law



- PD Control with Gravity Compensation (S&S, Ch.6.5.1)



Proper Inverse Dynamics Control

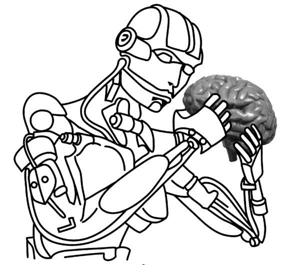


$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau$$

$$\mathbf{u} = \mathbf{B}(\mathbf{q})\ddot{\mathbf{q}}_{ref} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q}) + \mathbf{K}_D(\mathbf{q}_d - \mathbf{q})$$

- Note:
  - Online computation of inverse dynamics is always required
  - Higher derivatives of real system are required (not always possible with reasonable accuracy and acceptable phase lags)
  - Perfect knowledge of the dynamics model is assumed
  - High servo loop frequencies are required
  - Further improvements: Robust Control (S&S 6.5.3)

Operational Space Control



- Joint space:
    - Negative feedback control requires inverse kinematics, then joint space error can be computed
- Operational space:
    - Use direct kinematics to transform joint space variables into operational space, and compute operational space error (much cheaper computation)
  - New problems
    - Motor command in operational space needs to be transformed into joint space
    - System dynamics in operational space needs to be considered
    - Usually computationally expensive

Operational Space Control Schemes

- Jacobian Inverse Control (S&S 6.6.1)
- Jacobian Transpose Control (S&S 6.6.1)