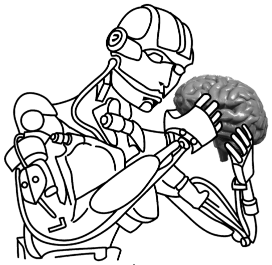
CS545—Contents IV

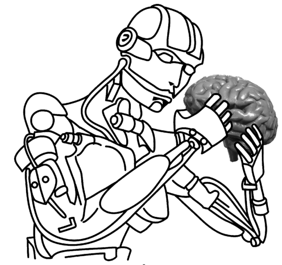
- Frequency Domain Representations
  - Laplace Transform
  - Most important Laplace Transforms
  - Transfer functions
  - Block-Diagram Algebra
  - Examples
- Matlab/Simulink Introduction
  - How to get started
  - The most relevant blocks and settings of Simulink
- Reading Assignment for Next Class
  - See http://www-clmc.usc.edu/~cs545

The Laplace Transform



- Properties of Frequency Domain Representations
  - A convenient method so solve (linear!) differential equations (even without a computer …) by converting them to algebraic equations
  - Makes system analysis easy, even for very big systems
  - Simple mathematics
  - Only applicable for linear time invariant systems!
  - Analyzes signals in terms of sinusoids and exponentials (includes Fourier transforms as special case)

Pierre-Simon, Marquis de Laplace 1749-1827
French Mathematician



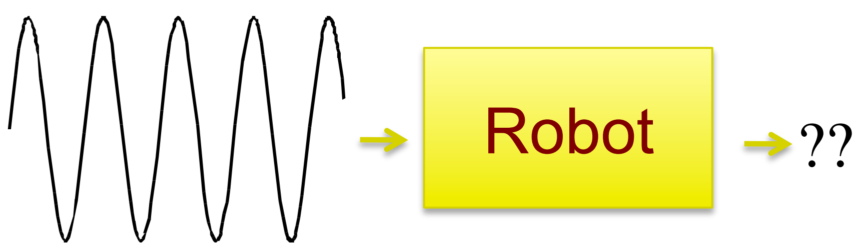
The Laplace Transform

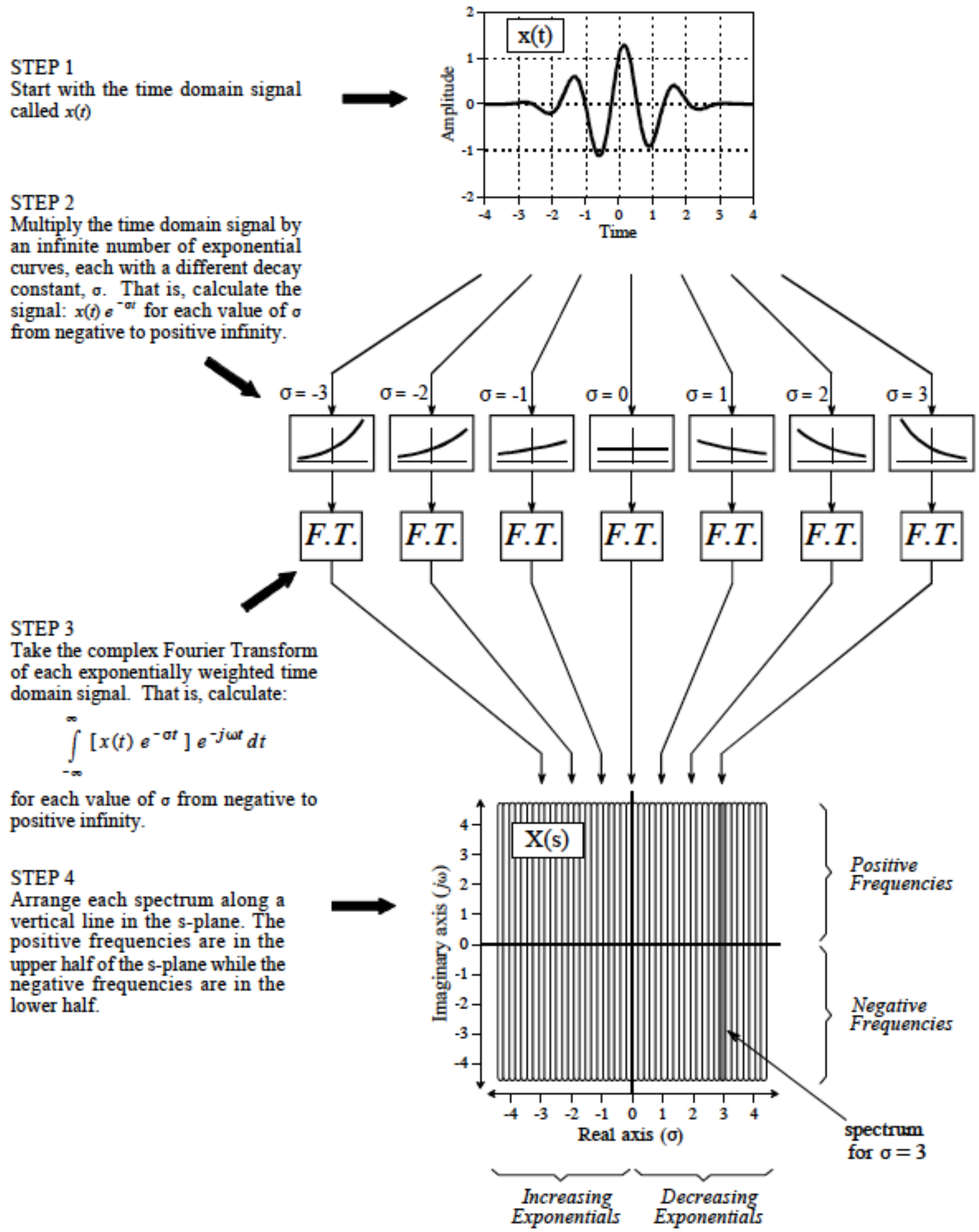
- The Core of Frequency Domain Analysis: The Laplace Transform

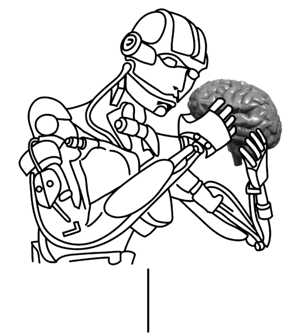
$$L\left(f(t)\right) = f(s) = \int_{0}^{\infty} f(t) e^{-st} dt$$

where

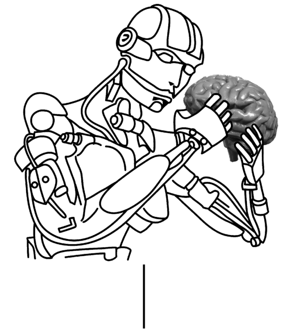
$$s = \sigma + j\omega \quad \text{and} \quad j = \sqrt{-1}$$

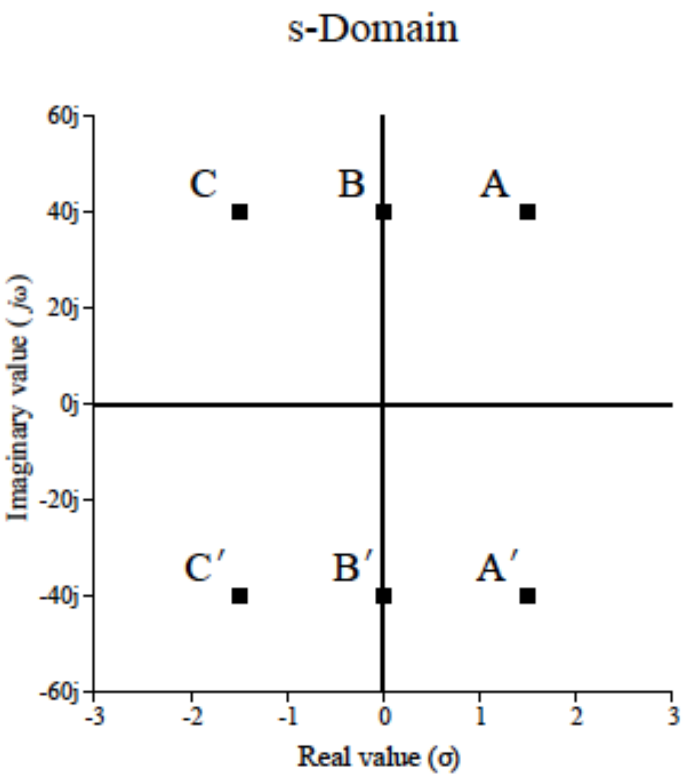






The Laplace transform. The Laplace transform converts a signal in the time domain, x(t) , into a signal in the s-domain, X(s) or X(**F**,**T**) . The values along each vertical line in the s-domain can be found by multiplying the time domain signal by an exponential curve with a decay constant **F**, and taking the complex Fourier transform. When the time domain is entirely real, the upper half of the s-plane is a mirror image of the lower half.





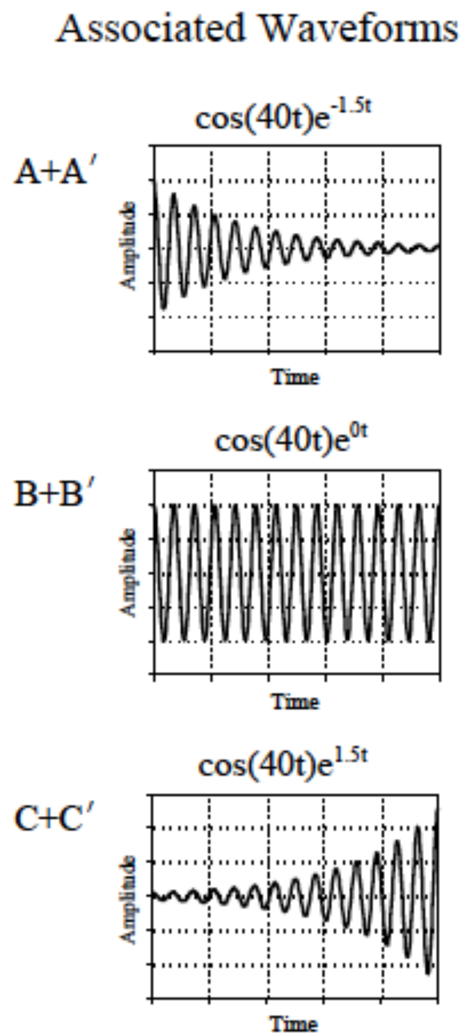
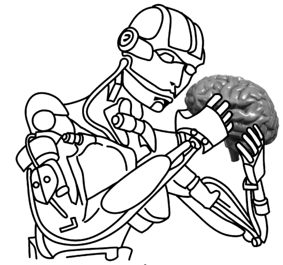


FIGURE 32-2
Waveforms associated with the s-domain. Each location in the s-domain is identified by two parameters: σ and ω. These parameters also define two waveforms associated with each location. If we only consider *pairs* of points (such as: A&A′, B&B′, and C&C′), the two waveforms associated with each location are sine and cosine waves of frequency ω, with an exponentially changing amplitude controlled by σ.

Most Important Laplace Transforms



$$L\left(ax(t)\right) = aL\left(x(t)\right) \quad \text{where } a \text{ is a constant}$$

$$L\left(x(t)\right) = x(s)$$

$$L\left(u(t)\right) = u(s)$$

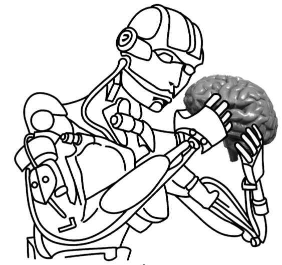
$$L\left(\dot{x}(t)\right) = sx(s) - x(0) \;\; \text{(commonly, } x(0) = 0 \text{ ,}$$

accomplished by coordinate transformations)

$$L\left(\ddot{x}(t)\right) = s^2 x(s) \quad \text{(and analogues for higher derivatives)}$$

$$L\left(\int x(t)dt\right) = \frac{1}{s}x(s)$$

Transfer Functions



- The Transfer Function describes the Input-Output Relationship of a dynamical system:

$$x(s) = H(s)u(s)$$

- Example I:

Time Domain:

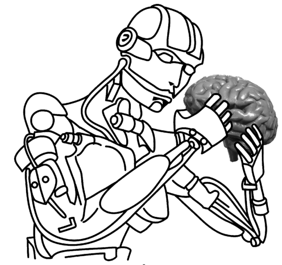
$$\ddot{x} = -b\dot{x} - kx + u$$

Frequency Domain:

$$s^2 x(s) = -bsx(s) - kx(s) + u(s)$$

$$x(s) = \frac{1}{s^2 + bs + k} u(s) = H(s)u(s)$$

Transfer Functions (cont' d)



- Example II: An Integrator

$$\dot{x} = u$$

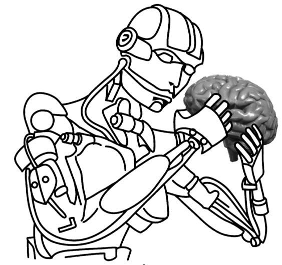
$$sx(s) = u(s) \quad \Rightarrow \quad x(s) = \frac{1}{s}u(s)$$

- Example III: A Simple Low Pass Filter

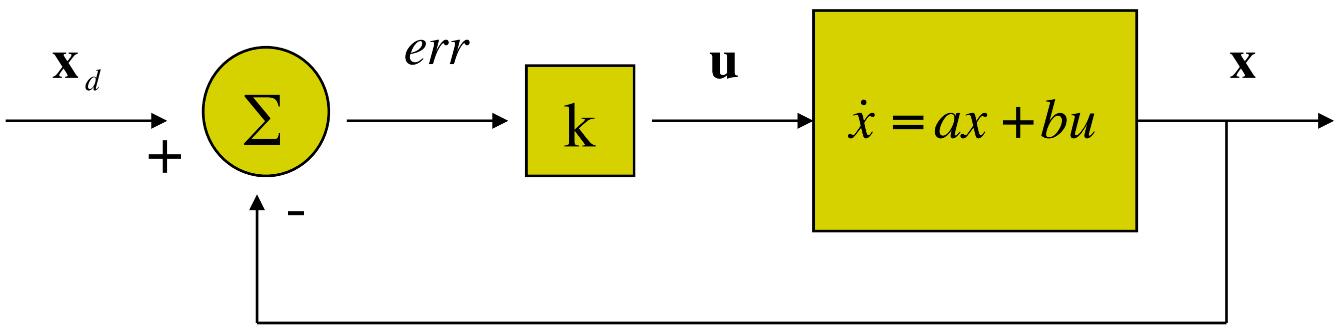
$$\dot{x} = \alpha(u - x)$$

$$sx(s) = -\alpha x(s) + \alpha u(s) \quad \Rightarrow \quad x(s) = \frac{\alpha}{s+\alpha}u(s)$$

Transfer Functions (cont' d)



- Example IV: A negative Feedback System

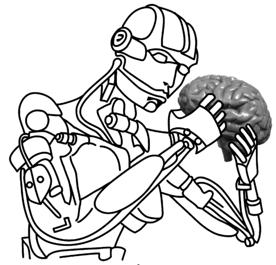


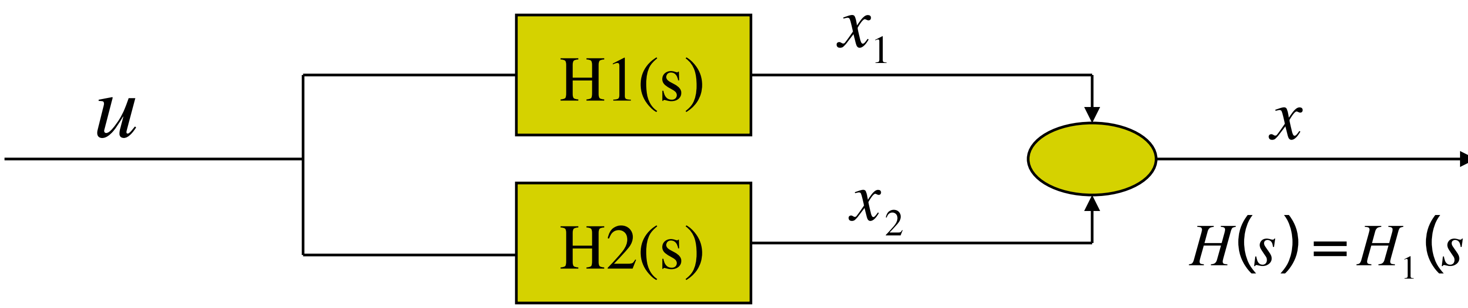
$$\dot{x} = ax + bu = ax + bk(x_d - x)$$

$$sx(s) = ax(s) + bk\left(x_d(s) - x(s)\right) = ax(s) + bkx_d(s) - bkx(s)$$

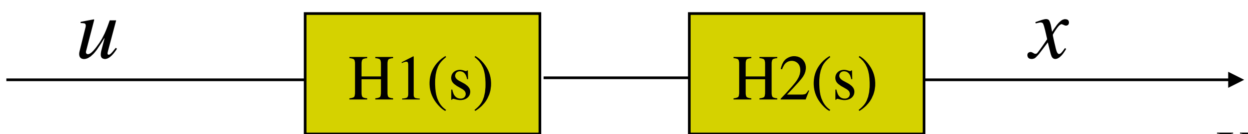
$$x(s) = \frac{bk}{s - a + bk} x_d(s)$$

Block Diagram Algebra

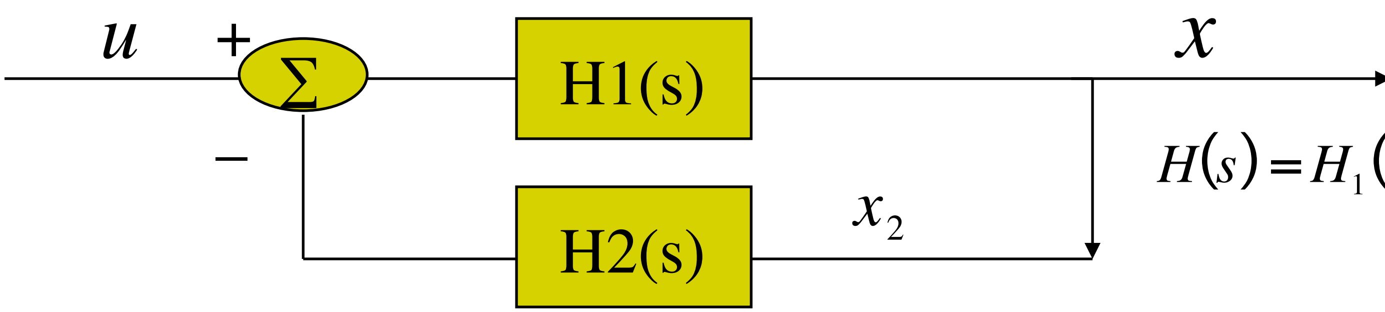




$$H(s) = H_1(s) + H_2(s)$$

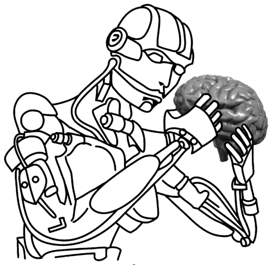


$$H(s) = H_2(s)H_1(s)$$



$$H(s) = H_1(s)(I + H_2(s)H_1(s))^{-1}$$

Matlab/Simulink Simulations



- An Example